Quiz 2 (*Sept. 25, Marks: 4, Duration: 30 mins*) **COL703, Jul-Nov 2023**

Name: **Entry No.:**

1. **[1 marks]** Let $X \subseteq \Phi$ and $\alpha \in \Phi$, where Φ denotes the set of propositional logic formulas. The use of strong completeness ($X \vDash \alpha$ *iff* $X \vdash \alpha$) can lead to an alternative, shorter, proof of compactness ($X \vDash \alpha$ *iff there exists* $Y \subseteq_{fin} X,\ Y \vDash \alpha$). Give that proof.

2. **[1 marks]** Show that if $\alpha \wedge \beta$ is consistent, then both α and β are consistent. Use Hilbert's proof system. Recall that α is said to be consistent if $\nvdash \neg\alpha$.

3. **[1 marks]** Consider the following popular puzzle. When asked for the ages of her three children, Mrs. Baker says that Alice is her youngest child if Bill is not her youngest child, and that Alice is not her youngest child if Carl is not her youngest child. Encode these facts, and the necessary background knowledge that only one of the three children can be her youngest child, into propositional logic formulas. Use propositions a, b and c to denote that Mrs. Baker's youngest child is Alice, Bill and Carl, respectively. Show with resolution that Bill is her youngest child.

4. **[1 marks]** Recall, from our discussion in the class, that Horn-SAT is in P. Consider the dual variant of the Horn satisfiability problem (Dual-Horn SAT) in which each clause has at most one negative literal. Is Dual-Horn SAT also in P? Why or why not?