Assignment 2 (*8 Marks, Due: 15 Oct*) **COL750, Jul-Dec 2024**

1. **[1 marks]** Assume $\Sigma = \{0, 1\}$. Let L be the language of ω-words over Σ that **do not** contain 01.
   - Give an ω-regular expression for L.
   - Give an NBA (Non-deterministic Büchi Automata) for L.

2. **[1 marks]** Suppose U is the regular language $a(a+b)^*a$. What is the NBA for U^ω?

3. **[2.5 marks]** The **F** operator in LTL is used to say that a property is true sometime in the *future*. Let us now introduce the **O** operator (short form for *Once*) to say that property was true sometime in the *past*.

   The formal semantics of **O** can be defined as follows. For an ω-word α, let α^i denote the suffix of α starting from the i^{th} position. Then:

   $\alpha^i \models \mathbf{O}\phi$ if $\exists\, j \leq i$ such that $\alpha^j \models \phi$, and $\alpha \models \mathbf{O}\phi$ if $\alpha^0 \models \mathbf{O}\phi$

   Let p_1 and p_2 be atomic propositions. Take the alphabet $\mathbb{B}^2 = \{\binom{0}{0}, \binom{0}{1}, \binom{1}{0}, \binom{1}{1}\}$ where the top element indicates the value for p_1 and the bottom one indicates the value of p_2. Let $\psi := \mathbf{G}(p_1 \rightarrow \mathbf{O}p_2)$.

   (a) Give two examples of ω-words over $\mathbb{B}^2$: one which satisfies ψ and one which does not satisfy ψ.

   (b) Show that ψ can be rewritten into an equivalent LTL formula which uses only the standard Until operator **U** and the boolean connectives $(\neg, \wedge, \vee, \rightarrow)$.

   (c) Construct a Non-deterministic Büchi Automata recognizing the language of ψ.

4. **[1.5 marks]** Let $\Sigma = \{a, b, c\}$. Construct a Büchi automata (deterministic or non-deterministic) for the following languages.

   (a) set of all ω-words where abc occurs at least once

   (b) set of all ω-words where abc occurs infinitely often

   (c) set of all ω-words where abc occurs finitely often

5. **[2 marks]** Let $\Sigma = \{a, b\}$. Define $L_{b \geq a} := \{\alpha \in \Sigma^\omega |$ in every finite prefix of α, the number of b's is $\geq$ the number of a's$\}$.

   (a) Give an example of an ω-word present in $L_{b \geq a}$.

   (b) Give an example of an ω-word which is not in $L_{b \geq a}$.

   (c) Is $L_{b \geq a}$ ω-regular? Justify.