COL750: Foundations of Automatic Verification (Jul-Dec 2024)

Lectures 05 & 06 (LTL: Syntax and Semantics)

Kumar Madhukar

madhukar@cse.iitd.ac.in

Aug 5th and 8th

Linear-time Temporal Logic

- has connectives that allow us to refer to the future
- models time as a sequence of states, extending infinitely into the future
- sequence of states is called a computational path
- since the future is not determined, we consider all possible paths

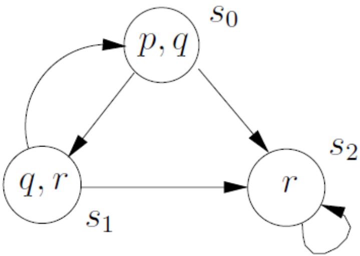
Syntax

See Sect. 3.2.1 of the Logic in Computer Science book by Huth and Ryan.

Syntax

- Well-formed formulas
- Binding priorities
- Parse trees
- Subformulas of an LTL formula

Semantics



Semantics

Let $\mathcal{M} = (\mathcal{S}, \rightarrow, \mathcal{L})$ be a model and $\pi = s_1 \rightarrow s_2 \rightarrow \ldots$ be a path in $\mathcal{M}$.

Whether π satisfies an LTL formula is defined by the satisfaction relation $\models$ as follows:

$$
\begin{aligned}
&\pi \models \top \\
&\pi \not\models \bot \\
&\pi \models p \text{ iff } p \in L(s_1) \\
&\pi \models \neg\phi \text{ iff } \pi \not\models \phi \\
&\pi \models \phi_1 \wedge \phi_2 \text{ iff } \pi \models \phi_1 \text{ and } \pi \models \phi_2 \\
&\pi \models \phi_1 \vee \phi_2 \text{ iff } \pi \models \phi_1 \text{ or } \pi \models \phi_2 \\
&\pi \models \phi_1 \rightarrow \phi_2 \text{ iff } \pi \models \phi_2 \text{ whenever } \pi \models \phi_1 \\
&\pi \models \mathrm{X}\,\phi \text{ iff } \pi^2 \models \phi \\
&\pi \models \mathrm{G}\,\phi \text{ iff, for all } i \geq 1,\ \pi^i \models \phi
\end{aligned}
$$

Semantics

Let $\mathcal{M} = (\mathcal{S}, \rightarrow, \mathcal{L})$ be a model and $\pi = s_1 \rightarrow s_2 \rightarrow \ldots$ be a path in $\mathcal{M}$.

Whether π satisfies an LTL formula is defined by the satisfaction relation $\vDash$ as follows:

$\pi \vDash \mathrm{F}\,\phi$ iff there is some $i \geq 1$ such that $\pi^i \vDash \phi$
$\pi \vDash \phi\,\mathrm{U}\,\psi$ iff there is some $i \geq 1$ such that $\pi^i \vDash \psi$ and for all $j = 1, \ldots, i-1$ we have $\pi^j \vDash \phi$

Semantics

Let $\mathcal{M} = (\mathcal{S}, \rightarrow, \mathcal{L})$ be a model and $\pi = s_1 \rightarrow s_2 \rightarrow \ldots$ be a path in $\mathcal{M}$.

Whether π satisfies an LTL formula is defined by the satisfaction relation $\vDash$ as follows:

$\pi \vDash \phi \mathrm{~W~} \psi$ iff either there is some $i \geq 1$ such that $\pi^i \vDash \psi$ and for all $j = 1, \ldots, i-1$ we have $\pi^j \vDash \phi$; or for all $k \geq 1$ we have $\pi^k \vDash \phi$

$\pi \vDash \phi \mathrm{~R~} \psi$ iff either there is some $i \geq 1$ such that $\pi^i \vDash \phi$ and for all $j = 1, \ldots, i$ we have $\pi^j \vDash \psi$, or for all $k \geq 1$ we have $\pi^k \vDash \psi$.

Example specifications

- it is impossible to get to a state where `started` holds, but `ready` does not hold

Example specifications

- for any state, if a `request` occurs, then it will eventually be granted

Example specifications

- a certain process is `enabled` infinitely often on every computational path

Example specifications

- on all paths, a certain process will eventually become `deadlocked`

Example specifications

- if a process is enabled infinitely often, then it runs infinitely often

Example specifications

- an upward travelling lift at the second floor does not change its direction when it has passengers wishing to go to the fifth floor

Example specifications

- the lift *can* remain idle on the third floor with its doors closed

Equivalences between LTL formulas

See Sect. 3.2.4 and Sect. 3.2.5 of the Logic in Computer Science book by Huth and Ryan.

Thank you!